# Written Exam for the M.Sc. in Economics Summer 2011 <br> Monetary Economics: Macro Aspects 

Master's Course

June 24
(3-hour closed-book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

This set contains three pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh $25 \%$ while question 3 weighs $50 \%$. These weights, however, are only indicative for the overall evaluation.

Henrik Jensen
Department of Economics, University of Copenhagen
Spring 2011

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.
(i) In the Calvo pricing model, expectations about future marginal costs are irrelevant for firms that by chance are able to reset their prices.
(ii) In a flex-price, cash-in-advance model, it is never possible to determine the optimal inflation rate.
(iii) In rational-expectations settings where real effects of monetary policy are due to unexpected policy movements, systematic components of monetary policy have no real effects.

## QUESTION 2:

## Money in the utility function: Finding money demand

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$
\begin{equation*}
U=\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}, m_{t}\right), \quad 0<\beta<1, \tag{1}
\end{equation*}
$$

with

$$
u\left(c_{t}, m_{t}\right) \equiv b \ln c_{t}+(1-b)\left(m^{F} \ln m_{t}-m_{t}\right), \quad 0<b<1, \quad m^{F}>0
$$

Agents maximize utility subject to the budget constraint

$$
\begin{align*}
c_{t}+k_{t}+m_{t} & =f\left(k_{t-1}\right)+\tau_{t}+(1-\delta) k_{t-1}+\frac{1}{1+\pi_{t}} m_{t-1}, \quad 0<\delta<1  \tag{2}\\
& \equiv \omega_{t},
\end{align*}
$$

where $c_{t}$ is consumption, $m_{t}$ is real money balances at the end of period $t, k_{t-1}$ is physical capital, $\tau_{t}$ are monetary transfers from the government, and $\pi_{t}$ is the inflation rate. Function $f$ satisfies $f^{\prime}>0, f^{\prime \prime}<0$.
(i) Derive the relevant first-order conditions for optimal behavior [Hint: Set up the value function $V\left(\omega_{t}\right)=\max _{c_{t}, m_{t}}\left\{u\left(c_{t}, m_{t}\right)+\beta V\left(\omega_{t+1}\right)\right\}$ and substitute out $\omega_{t+1}$ by (2) and $k_{t}$ by $\left.k_{t}=\omega_{t}-c_{t}-m_{t}\right]$
(ii) Interpret the first-order conditions intuitively, and show that they can be combined into

$$
\begin{equation*}
\frac{u_{m}\left(c_{t}, m_{t}\right)}{u_{c}\left(c_{t}, m_{t}\right)}=\frac{i_{t}}{1+i_{t}}, \tag{3}
\end{equation*}
$$

where $f_{k}\left(k_{t}\right)+1-\delta=\left(1+i_{t}\right) /\left(1+\pi_{t+1}\right)$ defines $i_{t}$ as the nominal interest rate. Discuss this and discuss whether steady-state superneutrality holds in the model.
(iii) Apply the particular functional form of $u$ and characterize the monetary policy that maximizes the utility of the representative agent and find the corresponding optimal steady-state real balances. Explain the results intuitively.

## QUESTION 3:

## Monetary policy trade offs

Consider the following log-linear model of a closed economy:

$$
\begin{align*}
& x_{t}=\mathrm{E}_{t} x_{t+1}-\sigma^{-1}\left(i_{t}-\mathrm{E}_{t} \pi_{t+1}-r_{t}^{n}\right), \quad \sigma>0,  \tag{1}\\
& \pi_{t}=\beta \mathrm{E}_{t} \pi_{t+1}+\kappa x_{t}, \quad 0<\beta<1, \quad \kappa>0, \tag{2}
\end{align*}
$$

where $x_{t}$ is the output gap, $i_{t}$ is the nominal interest rate (the monetary policy instrument), $\pi_{t}$ is goods price inflation and $r_{t}^{n}$ is the natural rate of interest, which is assumed to be a mean-zero, serially uncorrelated shock. $\mathrm{E}_{t}$ is the rational expectations operator conditional on all information up to and including period $t$.
(i) Discuss the economic mechanisms behind equations (1) and (2).
(ii) Assume that the monetary authority wants to minimize the loss function

$$
\begin{equation*}
L=\frac{1}{2} \mathrm{E}_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\lambda x_{t}^{2}+\pi_{t}^{2}\right], \quad \lambda>0 . \tag{3}
\end{equation*}
$$

Discuss the economic foundations for this loss function.
(iii) Derive the optimal values of $x_{t}$ and $\pi_{t}$ under discretionary policymaking [Hint: Consider $x_{t}$ the policy instrument, and acknowledge that under discretion the optimization problem becomes a sequence of static problems as expected values can be taken as given]. Discuss the solutions, and describe how the nominal interest rate will move with the natural rate of interest.
(iv) Assume that the monetary policymaker instead follows a rule for nominal interest-rate setting given as

$$
\begin{equation*}
i_{t}=\phi \pi_{t}, \quad \phi>1 . \tag{4}
\end{equation*}
$$

Derive the solutions for $x_{t}$ and $\pi_{t}$ for the system (1), (2) and (4). [Hint: Conjecture that the solutions are linear functions of the period's natural rate of interest, $r_{t}^{n}$, and remember that $\left.\mathrm{E}_{t} r_{t+1}^{n}=0.\right]$. Discuss the differences between these solutions and the ones obtained under discretionary policymaking. Can the monetary policy rule (4) be parameterized such that it will "deliver" the outcomes under discretionary policymaking?
(v) Will an ability to conduct optimal policy under commitment be advantegous in this setting? Why/Why not?

